

# การสร้างแผนภาพไบเฟอร์เคชันโดยใช้เทคนิคบงการเอ็มบีบนโปรแกรม PSCAD/EMTDC Bifurcation Diagram Generation with PSCAD/EMTDC and Multiple Poincaré Map

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## บทคัดย่อ

ไบเฟอร์เคชันไดอะแกรมเป็นเครื่องมือทางคณิตศาสตร์ที่สำคัญ สำหรับวิเคราะห์เฟอร์เรโซแนนซ์ในระบบไฟฟ้ากำลัง ไบเฟอร์เคชันไดอะแกรมแสดงให้เห็นถึงพฤติกรรมของระบบไม่เชิงเส้นอย่างชัดเจน วิธีการโดยทั่วไปสำหรับการสร้างไบเฟอร์เคชันไดอะแกรมในสถานะคงตัว จะเริ่มต้นจากการหาสมการอนุพันธ์ของระบบไฟฟ้าหนึ่งเฟสสมมูลอย่างง่าย และใช้เทคนิค continuation ช่วยหาผลเฉลย

บทความนี้จะประยุกต์ใช้โปรแกรม PSCAD/EMTDC และเทคนิค brute-force เพื่อสร้างไบเฟอร์เคชันไดอะแกรมของระบบไฟฟ้าสามเฟส โดยการใช้เครื่องมือใหม่ชื่อ “Multiple Poincaré Map” ที่สร้างขึ้นพร้อมกับเครื่องมือ File Read และ XY Plot ที่อยู่ในโปรแกรมจะได้ไบเฟอร์เคชันไดอะแกรม จากตัวอย่างทั้งสองที่ใช้ทดสอบเพื่อวิเคราะห์เฟอร์เรโซแนนซ์ในระบบไฟฟ้ากำลัง ได้แสดงให้เห็นว่าการประยุกต์ใช้โปรแกรม PSCAD/EMTDC และ “Multiple Poincaré Map” เพื่อสร้างไบเฟอร์เคชันไดอะแกรมให้ผลลัพธ์สอดคล้องกันกับผลการวัดในภาคสนาม

## Abstract

Bifurcation diagram is an important mathematical tool for the analysis of ferroresonance in the electrical power system. Bifurcation diagram can clearly present the behaviors of the nonlinear system. The general method for the generation of steady state bifurcation diagram is based on Ordinary Differential Equation (ODE) of the simplified single-phase circuit and the continuation technique.

This paper proposes the application of PSCAD/EMTDC simulation program and the brute-force technique to generate time response of the three-phase power system. A new device tool “Multiple Poincaré Map” is created and used with File Read and XY Plot to produce bifurcation diagrams with respect to changes of controlled parameters. Two examples related to ferroresonances in the distribution systems are presented. Bifurcation diagrams generated with the PSCAD/EMTDC and Multiple Poincaré Map confirm that the results from the simulation are in good agreement with the results from field tests.

**คำสำคัญ** : เฟอร์เรโซแนนซ์ ไบเฟอร์เคชันไดอะแกรม PSCAD/EMTDC XPPAUT

**Keywords** : Ferroresonance, Bifurcation Diagram, PSCAD/EMTDC, XPPAUT

## 1. Introduction

Ferroresonance is a nonlinear phenomenon, which exists in four modes [1]: 1. Fundamental mode, 2. Subharmonic mode, 3. Quasi-periodic mode, and 4. Chaotic mode. Different behaviors for each mode can be expressed by various tools such as bifurcation diagram, phase plane, Poincaré Section, etc. From the aspect of utility, bifurcation diagram is a convenient method for presenting the behavior of the steady responses with respect to the variation of controlled parameter in a wider range.

The ferroresonance phenomenon in electrical system can be modeled on the basis of Ordinary Differential Equation (ODE). This method is suitable for simple system (e.g. single phase circuit) which can be easily arranged into a simple dynamic equation. For the simplification process, some parameters have been neglected. This leads to compromise between accuracy and simplicity of the simulation. A software package for nonlinear dynamic calculation (e.g. XPPAUT) can be used to generate steady-state bifurcation diagrams. Application of XPPAUT with ODE method represents an excellent combination of software for the numerical analysis and for graphic presentation of nonlinear dynamic

systems.

Digital transient simulation program, PSCAD/EMTDC, is very popular for power system simulation due to the fact that equipment or subsystem can be individually modeled and connected together to represent a complex system without the need to simplify the system. Mathematical model for complex system can be created by simple model built up with the help of function blocks in the library. The outputs from the PSCAD/EMTDC simulation tool are in the form of temporal waveforms.

Another method for bifurcation diagram generation is based on brute-force technique with stroboscopic mapping [2]. This technique requires the solution of the differential equation for each set of parameters and operating condition. Each time series solution will be recorded using a stroboscopic mapping with the same sampling frequency as the frequency of the power supply. The Bifurcation diagram can be created from the plot of sampled point (or points) for each value of controlled parameter. Brute-force technique with stroboscopic mapping can be applied to both methods (ODE and PSCAD/EMTDC) of simulation.

Based on the same concept of bruteforce calculation and stroboscopic

mapping, a new device tool “Multiple Poincaré Map” is created with Fortran and PSCAD script languages. Combination of Multiple Poincaré Map with other PSCAD/EMTDC devices such as Multiple Run, File Read, XY plot and brute-force technique results in an extension of PSCAD/EMTDC for bifurcation diagram generation.

## 2. Research Methodology

### 2.1 Bifurcation Theory and Continuation Technique

Continuation technique is a mathematical technique in which an established solution of an equation follows a path in the space plane when a control parameter is varied. Based on the assumption that the solutions of an Ordinary Differential Equation (ODE) vary continuously with the initial conditions and the parameters of the ODE, a continuation algorithm and predictor-corrector scheme can trace the path of an established solution as the parameter varies. Ref. [3] explains and discusses the application of XPPAUT for bifurcation diagram generation in details, succinct information will be therefore given here.

The methodology of arc-length continuation requires at least one solution as a starting point. One of the methods to

determine this initial solution is to integrate the dynamic equations with respect to time until the steady state is reached. From the first solution, the predictor determines a likely new solution by linear extrapolation. The predicted solution is then corrected to the true solution by the corrector (e.g. by gradient descending algorithm). This extrapolation forms a corresponding solution branch. At intersections, where a solution generates two different solution branches, the algorithm can either resume continuation of the prevailing branch or perform branch switching to the intersecting one.

The application of continuation technique to bifurcation theory can be demonstrated by the solutions of the ferroresonant circuit.

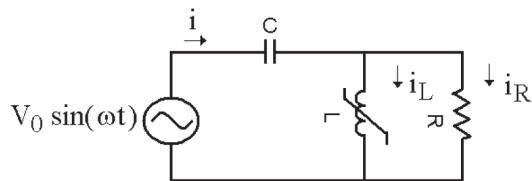


Fig. 1 Simple ferroresonant circuit

Fig. 1 shows a simple single-phase ferroresonant circuit. The behavior of this circuit can be described by the state equations with two state variables. The first state variable is the magnetic flux of the core ( $\phi_1$ ) and the second state variable is the derivative of the flux or the voltage ( $\phi_2 - \phi_1 = V$ ).

Based on Kirchhoff current and voltage laws and the assumption that the inductive current is a nonlinear function of the magnetic flux, the state equations for typical single-phase ferroresonant circuit is

$$\begin{aligned} \frac{d\phi_1}{d\tau} &= T_0\phi_2 \\ \frac{d\phi_2}{d\tau} &= T_0\left(-\left(\frac{1}{RC}\right)\phi_2 - \left(\frac{1}{C}\right)(a\phi_1 + b\phi_1^{11}) + V_0 \cos(\vartheta)\right) \end{aligned} \quad (1)$$

$$\frac{d\vartheta}{d\tau} = 2\pi$$

where  $\tau = t/T_0$ ,  $T_0 = 2\pi$ ,  $a=0.0028$ ,  $b = 0.0072$ ,  $C = 0.047F$  and  $R = 300 \Omega$

Fig. 2 shows the steady-state bifurcation diagram of the flux derivative ( $\phi_1$ ) with respect to changes in supply voltage amplitudes ( $V_0$ ) generated by the continuation technique. The time step was set at 0.01 sec. and initial conditions were  $\phi_1(0) = 0$ ,  $\phi_2(0) = 0$ , and  $\vartheta(0) = 0$ . It can be observed that Fold bifurcation occurred at 0.87 p.u. (LP 1) and 0.09 p.u. (LP 2) of supply voltage. Period-Doubling bifurcation occurred at the voltage above 1.63 p.u. (PD1). The waveform period at a new branch was twice in length of the previous one. When the new branch of Period-Doubling bifurcation continues to occur with increasing voltage, the unstable oscillations lead to chaotic mode. A chaotic mode (strange attractor) can be observed at 1.92 p.u. For this case study, Torus bifurcation did not occur.

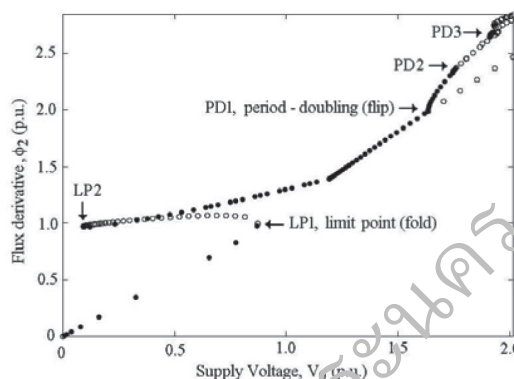


Fig. 2 The steady-state bifurcation diagram generated by the continuation technique with zero initial conditions

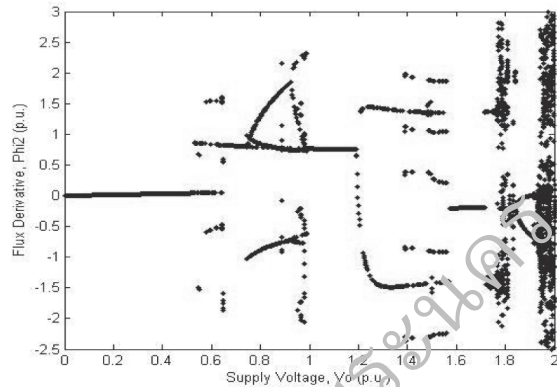
## 2.2 Bifurcation Diagram and Stroboscopic Mapping

Another technique for bifurcation diagram generation is to solve the differential equation for each set of parameter and operating condition. This technique is also called brute-force technique. Each resulting time series will be recorded by using a stroboscopic mapping with the sampling rate equal to the frequency of the supply voltage.

By increasing the amplitude of the supply gradually ( $V_0$ ) in small steps of 0.002 p.u. from 0 to 2.0 p.u. and sampling the solution in each step with the frequency of the supply voltage, a bifurcation diagram can be plotted as shown in Fig. 3. It is noted that stroboscopic mapping will generate only one point in the diagram if

the solution is in normal mode or fundamental mode of ferroresonance. The output value for normal mode and fundamental mode depend on the sampling instant, which can result in any value in a period of the output voltage. Multiple points for one value of power supply indicate unstable state. Chaotic mode can be clearly seen at the supply voltage close to 2.0 pu.

As shown in Fig.1, the bifurcation diagram for simple resonant circuit which was generated by stroboscopic mapping with different initial conditions ( $\phi_1(0) = 0$ ,  $\phi_2(0) = \sqrt{2}$  and  $\vartheta(0) = 0$ ) is also shown in Fig. 4. It can be seen that, for  $0 < V_0 < 0.12$  p.u., the solution is unique and corresponds to the normal state (no ferroresonance). For  $0.12 < V_0 < 0.53$  p.u., the solutions correspond to fundamental mode of ferroresonance and for  $0.53 < V_0 < 1.63$  p.u., there are two solutions: both fundamental and subharmonic ferroresonances. For  $1.63 < V_0 < 2.0$  p.u., the period doubling bifurcations occurred many times. Finally, the chaotic mode is observed near the voltage of 1.92 p.u.



**Fig. 3** The bifurcation diagram generated by stroboscopic mapping with the same sampling frequency as the frequency of the power supply

The correlation of bifurcation diagram in Fig. 2 can be made with those in Fig. 3 and Fig. 4. For Fig. 2, it shows three types of possible behaviors according to the value  $V_0$  from 0 to 0.87 p.u.. The dots on the curve between the origin and LP1 correspond to the normal state (without ferroresonance). The dots beyond LP2 correspond to the ferroresonance with fundamental mode (amplitude greater than 1 p.u.). The dots between LP1 and LP2 correspond to the unstable state. These three possible states depend on the initial conditions of the system. This can be seen in Fig. 3 that the system was in the normal state as the supply voltage was between 0 to 0.5 p.u. and in subharmonic state as it was from 0.5 to 0.62. However, in Fig. 4, the system

was in the normal state as the supply voltage was between 0 to 0.12 p.u. and in fundamental mode of ferroresonance for  $V_0$  was from 0.12 to 0.53 p.u.

XPPAUT software was used to simulate and plot both diagrams shown in Fig. 3 and Fig. 4. The results from these two figures show the quality of the information obtained from the bifurcation diagram.

### 2.3 Bifurcation Diagram Generation with PSCAD/EMTDC and Multiple Poincaré Map

The ferroresonant system described by the equation (1) can be simulated on PSCAD/EMTDC as shown in the diagram in Fig. 5.

A new custom device “Multiple Poincaré Map” for bifurcation diagram data file generation is shown in Fig. 6. Fortran and PSCAD script languages are used to program a custom device in PSCAD/EMTDC simulation program. Multiple Run device is used to vary the controlled parameter and it was named signal\_1. Terminals from signal\_2 to signal\_4 of the Multiple Poincaré Map are reserved to measure parameters. Period of time and range of record time can be set at appropriate values. The four parameters, from signal\_1 to signal\_4, can be saved

into a single file. The recorded data can be plotted on another PSCAD/EMTDC case file.

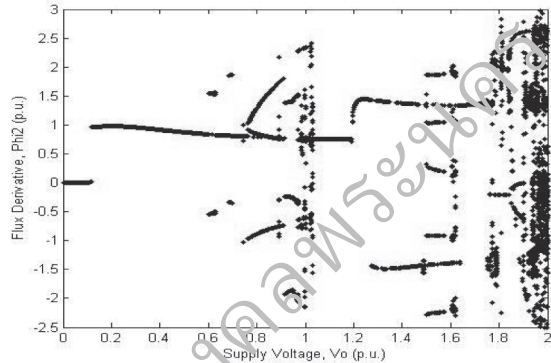


Fig. 4 The bifurcation diagram for simple resonant circuit generated by stroboscopic mapping with different initial conditions ( $\phi_1(0) = 0$ ,  $\phi_2(0) = \sqrt{2}$  and  $\vartheta(0) = 0$ )

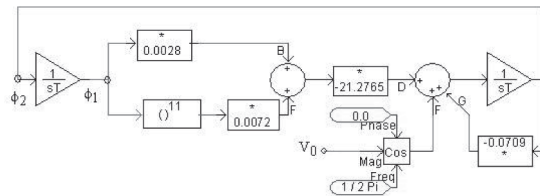


Fig. 5 Modeling of simple ferroresonant circuit with PSCAD/EMTDC

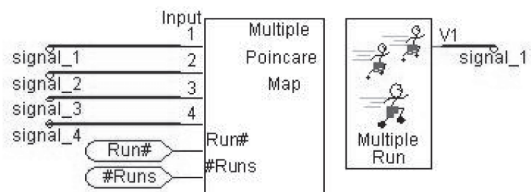


Fig. 6 Multiple Poincaré Map (PSCAD/EMTDC device tool) for bifurcation diagram generation



The flowchart of Multiple Poincaré Map is shown in Fig. 7. This Multiple Poincaré Map requires a complete solution in the form of system time response with each operating condition and each value of parameter. As shown in Fig. 8, when the solution reached steady state, the data was collected with the help of peak detection and Poincaré mapping. Only the peak of the waveform is recorded per period while the iterative calculation and data recording are made at each step of parameter change.

File Read and XY Plot (PSCAD/EMTDC device) shown in Fig. 9 are used to plot bifurcation diagrams. Bifurcation diagram is plotted on the plane where the ordinate is assigned for the measured parameter and the abscissa is assigned for the controlled parameter. Two operations are needed for Bifurcation diagram generation: the first operation involves simulation in order to generate data files, and the second operation is the plotting of a bifurcation diagram.

Fig. 10 shows the bifurcation diagram obtained from the PSCAD/EMTDC and Multiple Poincaré Map with the same conditions as in Fig. 4. The comparison between Fig. 4 and Fig. 10 shows good agreement of the results obtained from two techniques: ODE with XPPAUT and

PSCAD/EMTDC with Multiple Poincaré Map. With the aid of peak detection, the diagram with Multiple Poincaré Map offers clearer picture of the ferroresonance phenomenon. Especially, one can distinguish between normal mode (single point with normal amplitude), fundamental ferroresonance mode (single point with abnormal high amplitude), subharmonic mode (multiple points), and chaotic mode (many points appear in a range of voltages).

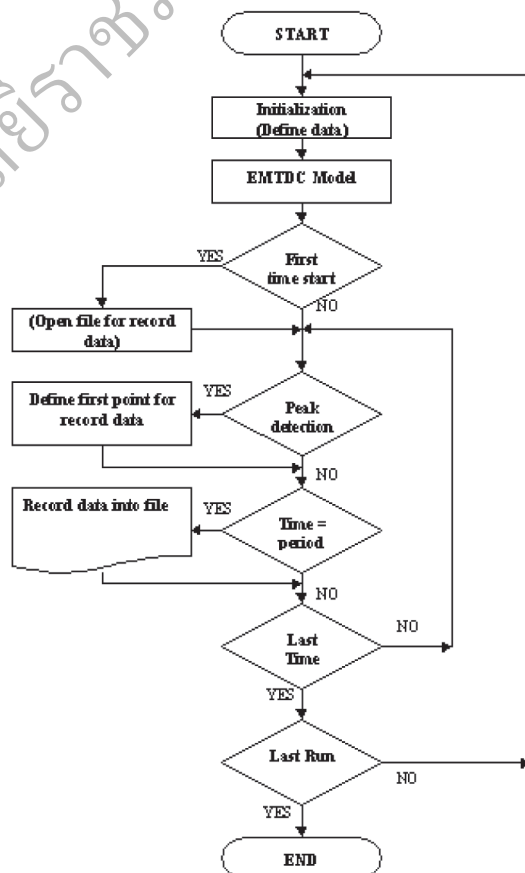
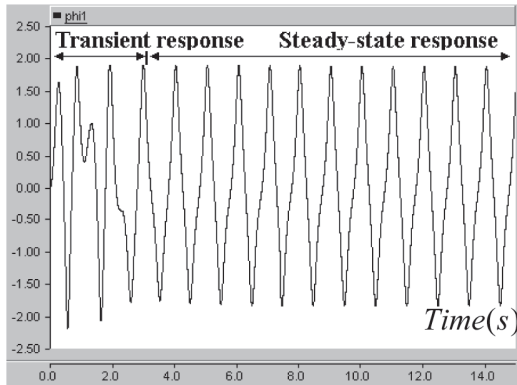
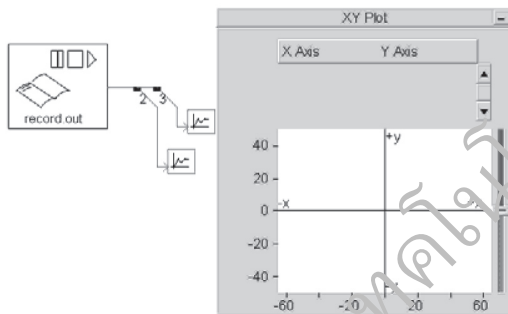


Fig. 7 Flowchart of Multiple Poincaré Map device tool

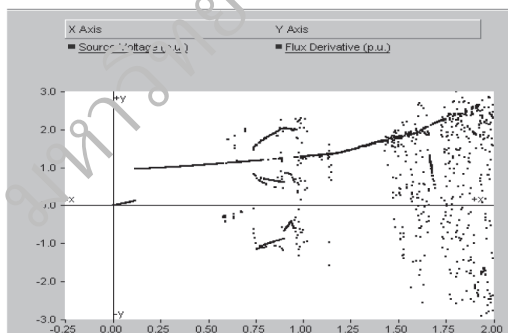
$\Phi(p.u.)$



**Fig. 8** Transient and steady-state responses of magnetic flux of the simple ferroresonant circuit



**Fig. 9** File Read with XY Plot (PSCAD/EMTDC device)



**Fig. 10** The bifurcation diagram with PSCAD/EMTDC and Multiple Poincaré Map for single-phase ferroresonant circuit

### 3. Results and Discussion

The digital transient analysis program, PSCAD/EMTDC is practical for three-phase power system simulation. The application of Multiple Poincaré Map device tool with PSCAD/EMTDC helps combine the advantages in term of the ease of simulation with one of the bifurcation diagram for the analysis of ferroresonance in three-phase power system.

Two studies of ferroresonance phenomena were carried out in Thailand for the Provincial Electricity Authority (PEA) [4] and the Metropolitan Electricity Authority (MEA) [5].

#### 3.1 Ferroresonance in a Distribution Network of PEA

The single line diagram of a the distribution network of Provincial Electric Authority (PEA) is shown in Fig. 11. It comprises four transformers of different sizes, i.e. 500 kVA, 100 kVA, and two transformers with the same rating of 50 kVA. The transformers are of three-phase type, 22/0.4 kV, with delta connection in the primary winding and wye-ground connection in the secondary winding (Dyn11). These transformers operated at no load and their cores were made of saturable iron cores. The no-load



switching was performed at the single-phase fused cutouts (SW1). The switching events consisted of single-phase sequential energization and de-energization (A-B-C and C-B-A, respectively). Four units of gapless metal oxide surge arresters were installed at the primary winding of each transformer.

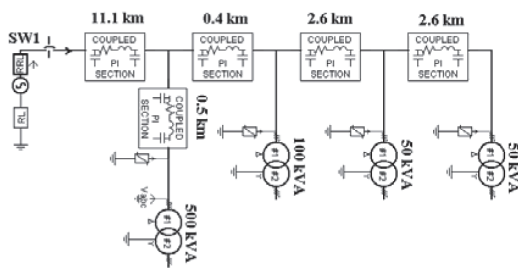


Fig. 11 Single line diagram of a PEA distribution network

The oscilogram resulted from the field test is shown in Fig. 12. De-energization on phase A led to the overvoltage phenomena with the magnitude of approximately 2.2 p.u. The recorded waveform shows chaotic ferroresonance mode.

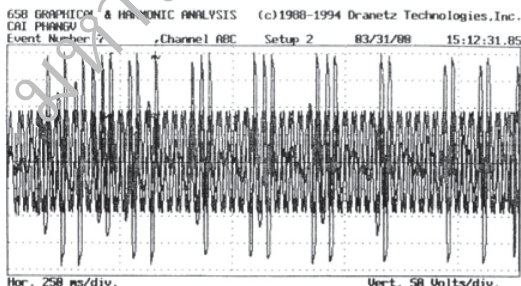


Fig. 12 The oscilogram of the field test after de-energization of phase A

The bifurcation diagram generated with PSCAD/EMTDC and Multiple Poincaré Map to investigate the effects of supply voltage on the transformer voltage is shown in Fig. 13. It can be seen that de-energization at the supply voltage greater than 1.0 p.u. will cause ferroresonance. The influence of the instant energization in the form of electrical angle of the voltage waveform at the time of switching is shown in Fig. 14. Although it is not possible to determine the exact switching instance in the field test due to the random operation, the simulation result shows that switching angles between 120 to 140 degrees and between 300 to 320 degrees can cause severe overvoltages.

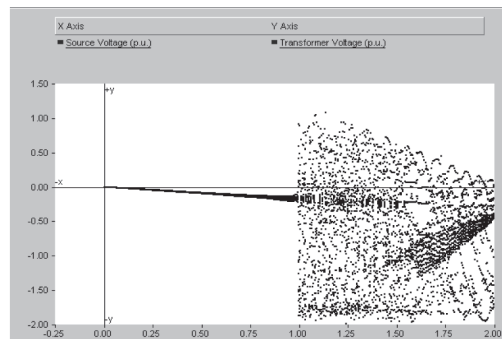


Fig. 13 The bifurcation diagram of transformer voltage with respect to the change in supply voltage

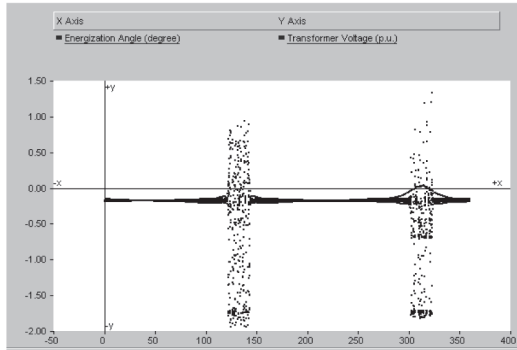


Fig. 14 The bifurcation diagram of transformer voltage with respect to the change in switching angle

### 3.2 Ferroresonance in a Distribution Network of MEA

The single line diagram of a part of Metropolitan Electricity Authority distribution system is shown in Fig. 15. Two distribution transformers, TR1 and TR2, are fed by MV feeder no. KMS 421. The test process was carried out by the opening of two drop-out fuses, F1 and F2. Then the breakers, A, B, C, RMU1 and RMU2 (all are three-phase switching), were switched on. The energizing process was started by closing the F2 fuses in single-phase sequence (C-B-A) operation. Immediately after the single-phase switching of F2, the surge arrester at phase A of distribution transformer (TR1) was damaged.

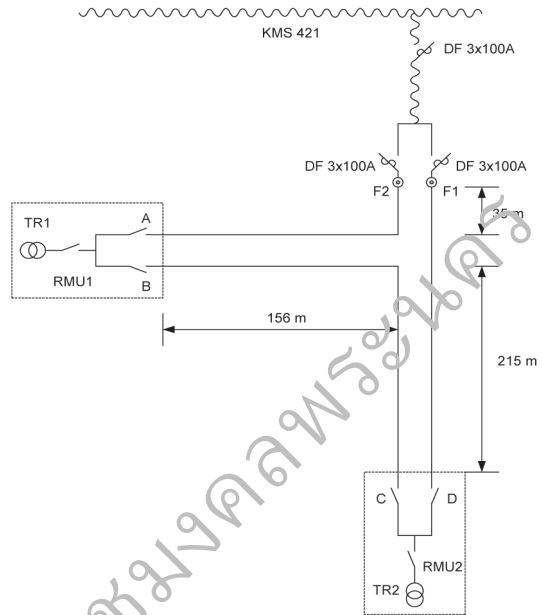
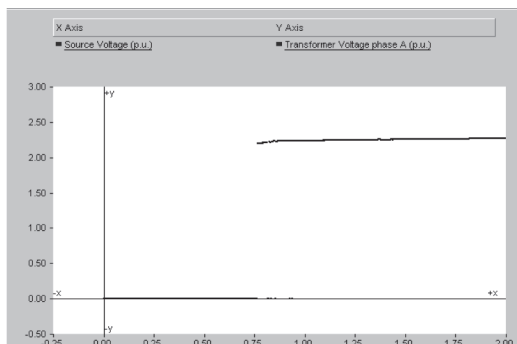


Fig. 15 Single line diagram of a part of MEA distribution system

The bifurcation diagram in Fig. 16 clearly shows that if the single-phase energization at F2 was performed at the voltage supply of 1 p.u., ferroresonance occurs at phase A. The overvoltage at 2 p.u. can damage the surge arrester as it was found in the field test. If the magnitude of supply voltage is less than 0.75 p.u., the overvoltage caused by ferroresonance can be avoided. Another mitigation technique is to open all switches first and then close the F2 fuses in single-phase sequence. After closing all contacts of F2, three-phase switch A, B, C, RMU1 and RMU2 can be closed without any risks of ferroresonance.



**Fig. 16** The bifurcation diagram of transformer voltage (phase A) with respect to the change in supply voltage for single-phase switching of F2

#### 4. Conclusions

Bifurcation diagrams generated from a simple ferroresonant circuit based on ODE, continuation technique and brute force technique with stroboscopic mapping were presented. For three phase power system simulation, PSCAD/EMTDC is more suitable due to the ease of modeling of individual equipment without the need for simplification. Individual equipment or subsystem can be connected to form a complex system. With brute force technique and a new device tool “Multiple Poincaré Map”, the time domain solutions obtained from PSCAD/EMTDC for various values of controlled parameters can be used to generate a bifurcation diagram.

Bifurcation diagrams generated from PSCAD/EMTDC and Multiple Poincaré

Map are very useful for the investigation of ferroresonance in three-phase power system. The effect of various operating conditions, controlled parameters and initial conditions can be easily investigated. Two examples of ferroresonance in the distribution systems show the advantage of ferroresonance investigation with bifurcation diagrams. The results of the simulation in the form of bifurcation diagrams can be used both to assess the risks of the power system to ferroresonance in the planning phase and to find the possible causes of damage to the equipment due to the ferroresonance.

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มหาวิทยาลัยเทคโนโลยีราชมงคลพระนคร